Numerical model of a compressible multi-fluid fluctuating flow

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Abstract

n th s nt q cons d th n m ca a o mat ons of m t- d com ss ct at ng o s Ass m ng that th o s com os d y non m ng com ss ds d d a mod zat on that can as an t ns on of th standa d com ss (k, ϵ) s mod s f ndam nta y n non cons at on fo m (th co ng t n ds and t nc n o s non cons at od cts) and th s a n t o m m thods fa non n a o ct on sch m s s d to s at th d sc t th ma n o t s of th mod n m ca com tat ons a fo m d on th chtm y -M sh o nsta ty to a dat th a oach and to m as th n nc of ct at ons

Key words com ss o s oc ty ct at ons non-cons at q at ons non n a o ct on m thods

1 Introduction

h n mod ng non m ng m t- d o on g n a y ass m that at a g n o nt of th doma n on y on com on nt s s nt fo a ag d a a s a a ays assoc at d to an n q com on nt of th d s d a s t at on s q d h n th d nt nt fac s a c t y cha act z d and t ac d Bas d on th s o s at on som n m ca m thods ha n d o d o 0

2 3 4 2 26 m a n d c t s n t s a o ac a t com t at ons of nt fac s/ nt fac s and nt fac s/ a s nt act ons o m com s c c a n t a many com nt fac s

n m ca a oac o os d n t s a do s not c t y c a act z o t ac t nt fac s nt fac s a a o m at d y a s t of c a act st c o- m s t d com on nts a s os d m d ys cs n t s o m s as to d n d n o d to a to od c co nt nt fac s/ nt fac s and nt fac s/ a s nt act ons fo cons d a s t of q at ons d d f om an ns m a ag ng d sc ng t a o of a m t-com on nt o n m of a a s n t s mod z at ons g o s t t n m of o com- on nts 23 s mod s os d n g n a t t ns o t to c nt on y fo o s t f com on nts Bas d on t ass m t on t at ss and oc ty a at ons a nstantan o s som s m d mod s a n d o d 24 8 25 n t s cas s t cts of oc ty ct at ons a not cons d d

g m n st gat d n t s a s non m d o at t ys ca mod and a y m d o at t n m ca mod n t s cont t som ass m t ons a nt od c d to d d a s m fy os d mod conta n ng a t man c a act st cs of t o as t s d a sco s cts On of t man ass m t o s t at oca y a t com on nts of t d a t sam a ag oc ty o t mod z at on ta s nto acco nt t d nc t n t a ag oc ty and t oc ty of ac com on nts fo t mod z at on ta s nto acco nt t oc ty ct at ons and nd t Bo ss n s q a o ma- t on t o m s os d Mo o som nt o y a anc q at ons a o ta n d and t y a st a d at t an s ng sco s ty m t

m ca m t od s d o d n t n t o m s f am o ys ca mod s d n t s nt o s go n d y non-cons at q at ons and som non c ass ca a o s a to cons d d 8 22 At t d sc t t s o t s a s d y a non n a o ct on fo m at on 2 3 So c s t ms a s t and nt g at d ana yt ca y o os d n m ca a oac s a dat d t t com t at on of t c t m y -M s o nt fac nsta ty

a s o gan z d as fo o s n t st s ct on t d at on of ys ca mod s o os d and t mat mat ca o t s a sta s d s cond s ct on s d ot d t t n m ca a o m at on n n m ca s ts a s nt d and ana yz d fo t conc s on

2 The physical model

L t s cons d a m t-com on nt o and ass m t at at cts ody fo c s and som d ss at on t ms can n g ct d s ng ns m a ag ng and ass man d d t fo o ng mod fo m t-com on nt o (ag s 26- 30)

$$\partial_t(\alpha_\ell\tilde{\rho}_\ell) + \nabla\cdot(\alpha_\ell\tilde{\rho}_\ell\mathbf{u}_\ell) = \quad \tilde{\rho}_\ell, \qquad (1)$$

$$\partial_t(\alpha_\ell\tilde{\rho}_\ell\mathbf{u}_\ell) + \nabla\cdot(\alpha_\ell\tilde{\rho}_\ell\mathbf{u}_\ell\otimes\mathbf{u}_\ell) = \nabla\cdot(\alpha_\ell(\sigma_\ell+\sigma'_\ell)) + \mathbf{u}_\ell, \qquad (2)$$

$$\partial_t(\alpha_\ell\tilde{\rho}_\ell \ \ell) + \nabla\cdot(\alpha_\ell\tilde{\rho}_\ell \ \ell\mathbf{u}_\ell) = \alpha_\ell\sigma_\ell \ \nabla\mathbf{u}_\ell + \alpha_\ell\tilde{\rho}_\ell\epsilon_\ell + \ \ell \qquad (3)$$

and

$$\alpha_\ell \tilde\rho_\ell \epsilon_\ell = -\partial_t(\alpha_\ell \tilde\rho_\ell k_\ell) - \nabla\cdot(\alpha_\ell\tilde\rho_\ell k_\ell \mathbf{u}_\ell) + \alpha_\ell \sigma'_\ell \quad \nabla \mathbf{u}_\ell + k_\ell, \tag{4}$$

h α_ℓ $\tilde\rho_\ell$ $\mathbf{u}_\ell$ ℓ and σ_ℓ a s ct y th a ag d o m f act on d ns ty oc ty nt na n gy and st ss t nso of th d com on nt n t c t - nt n gy of d com on nts s d not d k_ℓ h ϵ_ℓ d not s ts d ss at on at m an d ns ty $\tilde\rho_\ell$ s th mass of const t nt ℓ n t o m of const t nt ℓ notat on $\tilde\rho_\ell$ m st not conf s d th ρ_ℓ that d not s th a t a d ns ty (a so ca d th ct d ns ty of th com on nt ℓ)

od ct on of mass $\tilde\rho_\ell$ mom nt m $\mathbf{u}_\ell$ tota n gy ℓ and n t c t nt n gy k_ℓ a d n d y th t ansf at th nt fac s to cons at on o t s ha

$$\sum_\ell \tilde\rho_\ell = \sum_\ell \mathbf{u}_\ell = \sum_\ell \quad \ell = \sum_\ell k_\ell = 0 \tag{5}$$

h n th s no has t ans t on o ch m ca act on at nt fac s ha $\tilde\rho_\ell = 0$ s q at ons a o ta n d y nt od c ng a ct at on oc ty h ch s th d nc $\mathbf{u}'_\ell$ t n th com t d $\mathbf{v}$ and th m an d $\mathbf{u}_\ell$ n a s ntat o m

$$\mathbf{u}'_\ell = \mathbf{v} - \mathbf{u}_\ell, \tag{6}$$

h $\mathbf{u}_\ell$ s constant n th s ntat o m of th a ag ng a oach ct at on $\mathbf{u}'_\ell$ s d n d on y h th d com on nt ℓ s s nt n th yno ds st ss σ'_ℓ and th ct at on n t c n gy k_ℓ a assoc at d to $\mathbf{u}'_\ell$

n o d to d d a s m fy mod ass m that a at on oc ss s a nstantan o s s ch that th a ag d oc ty s th sam fo a com on nts

$$\mathbf{u}_\ell = \mathbf{u} \quad \text{fo a } \ell. \tag{ }$$

fo

$$\mathbf{u}'_\ell = \mathbf{u}', \quad k_\ell = k, \quad \epsilon_\ell = \epsilon \quad \text{fo a } \ell. \tag{8}$$

m an st ss t nso s a d n d y $\sigma_\ell = -p_\ell Id + \mu_\ell \tau(\mathbf{u})$ fo nd th Bo ss n sq a o mat on th yno ds st ss s t nd th fo m

$$\sigma'_\ell = -p'_\ell Id + \mu'_\ell \tau(\mathbf{u}) \quad \text{th} \quad p'_\ell = (\gamma' -)\tilde\rho_\ell k \tag{ }$$

h p'_ℓ s th s h ca a t of th t nso γ' s a constant ($\gamma' = \frac{5}{3}$ fo f ct on ss co s ons) and μ'_ℓ s th co c nt of ct at on scos ty Aft 3 (s a so 6) ado t th notat on γ' nst ad of 5/3 h ch ma s mo act ca s a com tat ons (fo nstanc s th mma 2 2)

S mm ng o a com on nts th mass th mom nt m and th ct at on n t c n gy o ta n

$$\begin{aligned}
\partial_t \rho + \nabla\cdot(\rho\mathbf{u}) &= \quad 0, && (0)\\
\partial_t(\rho\mathbf{u}) + \nabla\cdot(\rho\mathbf{u}\otimes\mathbf{u}) + \nabla(p+p') &= \nabla\cdot((\mu+\mu')\tau(\mathbf{u})), && ( )\\
\partial_t(\rho k) + \nabla\cdot(\rho k\mathbf{u}) + p'\nabla\cdot(\mathbf{u}) &= \mu'\tau(\mathbf{u}) \quad \nabla\mathbf{u} - \rho\epsilon, && (2)
\end{aligned}$$

h

$$\rho = \sum_\ell \alpha_\ell \tilde\rho_\ell, \quad p = \sum_\ell \alpha_\ell p_\ell, \quad p' = \sum_\ell \alpha_\ell p'_\ell, \tag{3}$$

$$\mu = \sum_\ell \alpha_\ell \mu_\ell, \quad \mu' = \sum_\ell \alpha_\ell \mu'_\ell.$$

As n t t nc mod zat on t o t on of t d ss at on at $\rho\epsilon$ s a o- mat d as fo o s

$$\partial_t(\rho\epsilon) + \nabla\cdot(\rho\epsilon\mathbf{u}) + \frac{2}{3}C_1\rho\epsilon\nabla\cdot(\mathbf{u}) = \mu''\tau(\mathbf{u}) \quad \nabla\mathbf{u} - R \qquad (14)$$

$$\mu'' = C_1\frac{\epsilon}{k}\mu', \quad R = C_2\rho\frac{\epsilon^2}{k}$$

C_1 and C_2 a mod ng constants n t ct at ons a at t t - nc som n m ca a s of t s constants can fo nd n o n a - s ntat o m of t mod t com on nts a so at d (n n a m c o-sca d sc t on (ag 00)) fo t mass f act on Y_ℓ t o m f act on α_ℓ t m an d ns ty $\tilde{\rho}_\ell$ and t a t a d ns ty ρ_ℓ a at d y t fo o ng at ons

$$Y_\ell = \frac{m_\ell}{m} = \frac{\tilde{\rho}_\ell V_\ell}{\rho V} = \frac{\tilde{\rho}_\ell}{\rho}\alpha_\ell \Longrightarrow \rho_\ell = \rho Y_\ell = \tilde{\rho}_\ell \alpha_\ell \qquad (15)$$

m and V a notat ons fo mass and o m ass m t at t a at on o- c ss s a nstantan o s (t sam m an oc ty) and t at t s no as t ans- s t on o c m ca act ons fo a $\ell = 0$ and t a anc of nt na n gy of ac com on nt t s as

$$\partial_t(\rho_\ell \ _\ell) + \nabla\cdot(\rho_\ell \ _\ell\mathbf{u}) + \alpha_\ell p_\ell\nabla\cdot(\mathbf{u}) = \alpha_\ell\mu_\ell\tau(\mathbf{u}) \quad \nabla\mathbf{u} + \rho_\ell\epsilon$$

d d mod s t n d sc d y t a anc q at ons

$$\begin{aligned}
\partial_t\rho + \nabla\cdot(\rho\mathbf{u}) &= 0, && (16)\\
\partial_t(\rho\mathbf{u}) + \nabla\cdot(\rho\mathbf{u}\otimes\mathbf{u}) + \nabla(p+p') &= \nabla\cdot((\mu+\mu')\tau(\mathbf{u})), && (1\ )\\
\partial_t(\rho_\ell \ _\ell) + \nabla\cdot(\rho_\ell \ _\ell\mathbf{u}) + \alpha_\ell p_\ell\nabla\cdot(\mathbf{u}) &= \alpha_\ell\mu_\ell\tau(\mathbf{u}) \quad \nabla\mathbf{u} + \rho_\ell\epsilon, && (18)\\
\partial_t(\rho k) + \nabla\cdot(\rho k\mathbf{u}) + p'\nabla\cdot(\mathbf{u}) &= \mu'\tau(\mathbf{u}) \quad \nabla\mathbf{u} - \rho\epsilon, && (19)\\
\partial_t(\rho\epsilon) + \nabla\cdot(\rho\epsilon\mathbf{u}) + \frac{2}{3}C_1\rho\epsilon\nabla\cdot(\mathbf{u}) &= C_1\tfrac{\epsilon}{k}\mu'\tau(\mathbf{u}) \quad \nabla\mathbf{u} - C_2\tfrac{\rho\epsilon^2}{k} && (20)
\end{aligned}$$

$p' = (\gamma' - 1)\rho k$ mod co s t s a q at on fo t tota n gy $E = \frac{1}{2}\rho\mathbf{u}^2 + \rho k + \sum_\ell \rho_\ell \ _\ell$ nd d f om (18) d d c

$$\partial_t(\sum_\ell \rho_\ell \ _\ell) + \nabla\cdot\left(\sum_\ell \rho_\ell \ _\ell\mathbf{u}\right) + p\nabla\cdot(\mathbf{u}) = \mu\tau(\mathbf{u}) \quad \nabla\mathbf{u} + \rho\epsilon. \qquad (21)$$

o f om (1) com t t o t on a of t n t c n gy s t

$$\mathbf{u}\cdot\Big(\partial_t(\rho\mathbf{u}) + \nabla\cdot(\rho\mathbf{u}\otimes\mathbf{u}) + \nabla(p+p')\Big) = \mathbf{u}\cdot\Big(\nabla\cdot\big((\mu+\mu')\tau(\mathbf{u})\big)\Big). \qquad (22)$$

Sinc ha

$$
\begin{aligned}
\mathbf{u}\cdot\partial_t\rho\mathbf{u} &= \rho\partial_t\frac{\mathbf{u}^2}{2}+\mathbf{u}^2\partial_t\rho,\\
&= \partial_t\rho\frac{\mathbf{u}^2}{2}-\mathbf{u}^2\nabla\cdot(\rho\mathbf{u}),
\end{aligned}
$$

and

$$\mathbf{u}\cdot\nabla\cdot(\rho\mathbf{u}\otimes\mathbf{u})=\nabla\cdot\left(\rho\frac{\mathbf{u}^2}{2}\mathbf{u}\right)+\frac{\mathbf{u}^2}{2}\nabla\cdot(\rho\mathbf{u}),$$

th q at on (22) ads as fo o s

$$\partial_t\left(\rho\frac{\mathbf{u}^2}{2}\right)+\nabla\cdot\left(\rho\frac{\mathbf{u}^2}{2}\mathbf{u}\right)+\mathbf{u}\nabla(p+p')=\mathbf{u}\cdot\nabla\cdot\big((\mu+\mu')\tau(\mathbf{u})\big). \quad (23)$$

s m (o) (2) and (23) to o ta n

$$\partial_t E+\nabla\cdot\left((E+p+\frac{2}{3}\rho k)\mathbf{u}\right)=\mathbf{u}\cdot\nabla\cdot\big((\mu+\mu')\tau(\mathbf{u})\big)+(\mu+\mu')\tau(\mathbf{u})\quad\nabla\mathbf{u}.$$

Ass m that $\tau(\mathbf{u})$ s symm t c to t

$$\mathbf{u}\cdot\nabla\cdot\big((\mu+\mu')\tau(\mathbf{u})\big)+(\mu+\mu')\tau(\mathbf{u})\quad\nabla\mathbf{u}=\nabla\cdot\big((\mu+\mu')\tau(\mathbf{u})\mathbf{u}\big).$$

s a cons at on of th tota n gy s th s o ta n d

$$\partial_t E+\nabla\cdot\left((E+p+\frac{2}{3}\rho k)\mathbf{u}\right)=\nabla\cdot\big((\mu+\mu')\tau(\mathbf{u})\mathbf{u}\big) \quad (24)$$

n o d to o ta n a os d o m n d mo c os ass m t ons

2.1 Closure assumptions and mathematical properties

L t s cons d n th s s ct on th fo o ng fo m at on of th o s syst m fo a m t of n_p com on nts

$$
\left\{
\begin{array}{l}
\partial_t\rho+\partial_x\rho u=0,\\
\partial_t(\rho u)+\partial_x(\rho u^2+p+\frac{2}{3}\rho k)=\partial_x((\mu+\mu')\partial_x u),\\
\partial_t(\rho_\ell e_\ell)+\partial_x(\rho_\ell e_\ell u)+\tilde{p}_\ell\partial_x u=\tilde{\mu}_\ell(\partial_x u)^2+\rho_\ell\epsilon,\quad 1\le\ell\le n_p,\\
\partial_t(\rho k)+\partial_x(\rho k u)+\frac{2}{3}\rho k\partial_x u=\mu'(\partial_x u)^2-\rho\epsilon,\\
\partial_t(\rho\epsilon)+\partial_x(\rho\epsilon u)+\frac{2}{3}C_1\rho\epsilon\partial_x u=C_1\frac{\epsilon}{k}\mu'(\partial_x u)^2-C_2\rho\frac{\epsilon^2}{k},
\end{array}
\right. \quad (25)
$$

h $\tilde{p}_\ell=\alpha_\ell p_\ell$ and $\tilde{\mu}_\ell=\alpha_\ell\mu_\ell$

L t s d not y s_ℓ th s c c nt o y of a g n com on nt s cond a of th th modynam c a s fo ach com on nt of th d s tt n as

$$de_\ell=-T_\ell ds_\ell-p_\ell dv_\ell, \quad (26)$$

h $T_\ell>0$ s th t m at and v_ℓ s th s c c o m n th s nt o s_ℓ s th math mat ca nt o y nst ad of $-s_\ell$ h ch d not s th hys ca nt o y (s od s and a at 5 to f th d ta s) As s a th f nct ons $(v_\ell, s_\ell) \rightarrow e_\ell(v_\ell, s_\ell)$ a ass m d to st ct y con and sat sfy

$$\frac{\partial e_\ell}{\partial v_\ell}(v_\ell, s_\ell) = -p_\ell < 0 \quad \text{and} \quad \frac{\partial e_\ell}{\partial s_\ell}(v_\ell, s_\ell) = -T_\ell < 0. \tag{2 }$$

Acco d ng to th mod ng ass m t ons o os d n 6 th th modynam cs a com t d y th at ons

$$\begin{cases} D_t v_\ell &= \lambda_\ell D_t v \\ D_t \lambda_\ell &= 0 \end{cases} \tag{28}$$

h $\lambda_\ell > 0$ s a g n s t of a am t s and $D_t\phi = \partial_t\phi + \mathbf{u}\partial_x\phi$ s th mat a d at of th q ant ty ϕ

LEMMA 2.1 h n th ass m t ons (28) a cons d d th $\lambda_\ell = \frac{\rho}{\tilde\rho_\ell} = \frac{\alpha_\ell}{Y_\ell}$ smooth so t ons of (25)–(28) sat sfy th fo o ng nt o y n q a t s

$$\partial_t \rho s_\ell + \partial_x \rho s_\ell u = -\frac{\lambda_\ell \mu_\ell}{T_\ell}(\partial_x u)^2 - \frac{\rho\epsilon}{T_\ell} \leq 0, \quad \ell = , n_p. \tag{2 }$$

As a cons q nc th fo o ng nt o y a anc q at ons a o ta n d

$$\frac{\beta_{n_p}}{T_{n_p}}\left\{\partial_t \rho s_\ell + \partial_x \rho s_\ell u\right\} - \frac{\beta_\ell}{T_\ell}\left\{\partial_t \rho s_{n_p} + \partial_x \rho s_{n_p} u\right\} = \frac{\rho\epsilon}{T_\ell T_{n_p}}\left(\beta_\ell - \beta_{n_p}\right), \tag{30}$$

fo $\leq \ell \leq n_p -$ th $\beta_\ell = \dfrac{\lambda_\ell \mu_\ell}{\sum_k \lambda_k \mu_k}$

oof d nt ty (2) s o ta n d f om th at on

$$\rho Y_\ell D_t e_\ell + \alpha_\ell p_\ell \partial_x u = \alpha_\ell \mu_\ell (\partial_x u)^2 + \rho_\ell \epsilon.$$

s ng (2) and (5) th s at on t s as

$$-\rho Y_\ell p_\ell D_t v_\ell - \rho Y_\ell T_\ell D_t s_\ell + \lambda_\ell Y_\ell p_\ell \partial_x u = \lambda_\ell Y_\ell \mu_\ell (\partial_x u)^2 + \rho_\ell \epsilon.$$

By th ass m t ons ha $D_t v_\ell = \lambda_\ell D_t(/\rho) = \frac{\lambda_\ell}{\rho}\partial_x u$ fo

$$-\rho Y_\ell T_\ell D_t s_\ell = \lambda_\ell Y_\ell \mu_\ell (\partial_x u)^2 + \rho Y_\ell \epsilon.$$

n (2) s o d and (30) fo o s □

nt o y a anc q at ons ha sta sh d n th a o s t a d ot d to ach d n fact a s m a s t ho ds t conc n ng th t nc 3 6 nd d ha

LEMMA 2.2 smooth so t ons of (25)–(28) sat sfy th fo o ng t nt n- t o y at on

$$\partial_t \rho s' + \partial_x \rho s' u = \frac{\gamma' - }{\rho^{\gamma'-1}}\left(\mu'(\partial_x u)^2 - \rho\epsilon\right) \quad \text{th} \quad s' = \frac{(\gamma' -)\rho k}{\rho^{\gamma'}} \tag{3 }$$

As a cons q nc th fo o ng nt o y a anc q at ons a o ta n d

$$\mu' \frac{\gamma'-1}{\rho^{\gamma'-1}} \left\{\partial_t \rho s_{n_p} + \partial_x \rho s_{n_p} u\right\} - \frac{\lambda_{n_p}\mu_{n_p}}{T_{n_p}} \left\{\partial_t \rho s' + \partial_x \rho s' u\right\} = \frac{\rho\epsilon}{T_{n_p}} \frac{\gamma'-1}{\rho^{\gamma'-1}} (\mu' - \lambda_{n_p}\mu_{n_p}). \quad (32)$$

oof o ta n th d nt ty (31) t s t th o t on a of ρk as fo o s

$$\frac{(\gamma'-1)}{\rho^{\gamma'}} \left(\partial_t(\rho k) + u\partial_x(\rho k) + \gamma' \rho k \partial_x u\right) = \frac{(\gamma'-1)}{\rho^{\gamma'}} \left(\mu'(\partial_x u)^2 - \rho\epsilon\right). \quad (33)$$

o f om th cont n ty a ha

$$(\gamma'-1)\rho k \left(\partial_t \frac{1}{\rho^{\gamma'}} + u\partial_x \frac{1}{\rho^{\gamma'}} - \frac{\gamma'}{\rho^{\gamma'}} \partial_x u\right) = 0. \quad (34)$$

n (33) and (34) g

$$\partial_t s' + u\partial_x s' = \frac{\gamma'-1}{\rho^{\gamma'}} \left(\mu'(\partial_x u)^2 - \rho\epsilon\right).$$

s ng th at on $\rho\left(\partial_t s' + u\partial_x s'\right) = \partial_t \rho s' + \partial_x \rho s' u$ th at on (31) s o ta n d q at on (32) s o ta n d y com n ng (24) and (31) □

om th syst m (25) t s y asy to d th fo o ng q at on

$$\partial_t \frac{k^{C_1}}{\epsilon} + u\partial_x \frac{k^{C_1}}{\epsilon} = (C_2 - C_1) k^{C_1 - 1}. \quad (35)$$

n th s q th a a $\frac{k^{C_1}}{\epsilon}$ s d nst ad of $\rho\epsilon$

s mma z th n gy and th nt o y a anc q at ons (s L mma 21) t a so th add t ona t nt o t on a s (s L mma 22) a s d to fo m at th non cons at syst m (25) nd th fo o ng q a nt fo m

$$\begin{cases} \partial_t \rho + \partial_x \rho u = 0, \\ \partial_t \rho u + \partial_x (\rho u^2 + p + \frac{2}{3}\rho k) = \partial_x ((\mu + \mu')\partial_x u), \\ \partial_t E + \partial_x (E + p + \frac{2}{3}\rho k) u = \partial_x ((\mu + \mu') u \partial_x u), \\ \frac{\beta_{n_p}}{T_{n_p}} \left\{\partial_t \rho s_\ell + \partial_x \rho s_\ell u\right\} - \frac{\beta_\ell}{T_\ell} \left\{\partial_t \rho s_{n_p} + \partial_x \rho s_{n_p} u\right\} = \frac{\rho\epsilon}{T_\ell T_{n_p}} (\beta_\ell - \beta_{n_p}), \\ \frac{\lambda_{n_p}\mu_{n_p}}{T_{n_p}} \left\{\partial_t \rho s' + \partial_x \rho s' u\right\} - \mu' \frac{\gamma'-1}{\rho^{\gamma'-1}} \left\{\partial_t \rho s_{n_p} + \partial_x \rho s_{n_p} u\right\} = \\ \qquad \frac{\rho\epsilon}{T_{n_p}} \frac{\gamma'-1}{\rho^{\gamma'-1}} (\lambda_{n_p}\mu_{n_p} - \mu'), \\ \partial_t \rho \frac{k^{C_1}}{\epsilon} + \partial_x \rho \frac{k^{C_1}}{\epsilon} u = (C_2 - C_1)\rho k^{C_1 - 1}, \end{cases} \quad (36)$$

th $1 \leq \ell \leq n_p - 1$ Acco d ng to th nt o y a anc q at on t s st m has z that s_{n_p} s not an n no n of (36) t t ns o t to a f nct on of th n no ns $s_{n_p} = s_{n_p}(\rho, \rho u, E, \rho s_1, ..., \rho s_{n_p - 1}, \rho s', \rho k^{C_1}\epsilon)$

L t s ass m that th scos ty f nct ons a a od ct of th cha act st c scos ty of th has y a f nct on of th a t a t m at cons d th fo o ng a t a y cho c

$$\lambda_{n_p}\mu_\ell = (Y_\ell T_\ell)^{m_\ell}\bar{\mu}_\ell \quad \leq \ell \leq n_p,$$

h $m_\ell > 0$ a a constants to d n h n th a t a t m at s a g n ($\tilde{T}_\ell = Y_\ell T_\ell = \tilde{p}_\ell v/(\gamma_\ell -)$) th a a s β_ℓ can com t d as fo o s

$$\beta_\ell = \frac{(\tilde{T}_\ell)^{m_\ell}\bar{\mu}_\ell}{\sum\limits_{1\leq l\leq n_p}(\tilde{T}_l)^{m_l}\bar{\mu}_l}. \tag{3 }$$

ach β_ℓ t ns o t to a s t f nct on cha act z ng mat a nt fac s

cons at a a $\mathbf{w}^C$ th assoc at d $\mathbf{f}(\mathbf{w})$ th d s on $D(\mathbf{w})$ and th so c S^C t ms a d n d y

$$\mathbf{w}^C = \begin{pmatrix} \rho \\ \rho u \\ E \\ \rho Y_1 \\ \frac{\rho k^{C_1}}{\epsilon} \end{pmatrix}, \quad \mathbf{f}(\mathbf{w}) = \begin{pmatrix} \rho u \\ \rho u^2 + p + p' \\ (E + p + p')u \\ \rho Y_1 u \\ \frac{\rho k^{C_1}}{\epsilon}u \end{pmatrix},$$

$$D(\mathbf{w}) = \begin{pmatrix} 0 \\ \partial_x((\mu+\mu')\partial_x u) \\ \partial_x((\mu+\mu')u\partial_x u) \\ 0 \\ 0 \end{pmatrix}, \quad S^C(\mathbf{w}) = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ (C_2 - C_1)\rho k^{C_1 - 1} \end{pmatrix},$$

h $\mathbf{w} = {}^t(\mathbf{w}^C, \mathbf{w}^{NC})$ cto of non cons at a a s $\mathbf{w}^{NC}$ and th assoc at d $\mathbf{g}(\mathbf{w})$ th so c t m $S^{NC}(\mathbf{w}, \rho s_{n_p})$ and a cto $Q(\mathbf{w}, \rho s_{n_p})$ a d n d y

$$\mathbf{w}^{NC} = \begin{pmatrix} \rho s_1 \\ \\ \rho s_{n_p-1} \\ \rho s' \end{pmatrix}, \quad \mathbf{g}(\mathbf{w}) = \begin{pmatrix} \rho s_1 u \\ \\ \rho s_{n_p-1} u \\ \rho s' u \end{pmatrix},$$

$$Q(\mathbf{w}, \rho s_{n_p}) = \begin{pmatrix} \frac{\beta_1 T_{n_p}}{T_1 \beta_{n_p}} \\ \\ \frac{\beta_{n_p-1} T_{n_p}}{T_{n_p-1}\beta_{n_p}} \\ \frac{\mu' T_{n_p}(\gamma'-1)}{\lambda_{n_p}\mu_{n_p}\rho^{\gamma'-1}} \end{pmatrix}, \quad S^{NC}(\mathbf{w}, \rho s_{n_p}) = \begin{pmatrix} \frac{\rho\epsilon(\beta_1 - \beta_{n_p})}{T_1\beta_{n_p}} \\ \\ \frac{\rho\epsilon(\beta_{n_p-1} - \beta_{n_p})}{T_{n_p-1}\beta_{n_p}} \\ \frac{\rho\epsilon}{\lambda_{n_p}\mu_{n_p}(\lambda_{n_p}\mu_{n_p} - \mu')}\frac{\gamma'-1}{\rho^{\gamma'-1}} \end{pmatrix}.$$

fo th mod t s as

$$\begin{cases} \partial_t \mathbf{w}^C + \partial_x \mathbf{f}(\mathbf{w}) = D(\mathbf{w}) + S^C(\mathbf{w}), \\ \partial_t \mathbf{w}^{NC} + \partial_x \mathbf{g}(\mathbf{w}) = S^{NC}(\mathbf{w}, \rho s_{n_p}) + Q(\mathbf{w}, \rho s_{n_p})\left\{\partial_t \rho s_{n_p} + \partial_x \rho s_{n_p} u\right\}. \end{cases} \tag{38}$$

L t s not that th st o d t act d syst m g n y

$$\begin{cases} \partial_t \mathbf{w}^C + \partial_x \mathbf{f}(\mathbf{w}) = 0 \\ \partial_t \mathbf{w}^{NC} + \partial_x \mathbf{g}(\mathbf{w}) = Q(\mathbf{w}, \rho s_{n_p}) \left\{ \partial_t \rho s_{n_p} + \partial_x \rho s_{n_p} u \right\}, \end{cases}$$

s hy o c g n a s a

$$u \pm c, \quad u, \quad \text{th} \quad c^2 = \sqrt{\frac{\gamma p + \gamma' p'}{\rho}}.$$

g n a s $u \pm c$ a on o d of m t cty h th g n a u s $n_p + 3$ o d of m t c ty Acco d ng to th o s 4 6 on can o th st nc of t a ng a so t ons s so t ons a s f to o os a d n t on of shoc a so t ons of th non-cons at hy o c syst m (s 8 o 4 6) s s not th os of th s nt o and foc s o att nt on on th n m ca a o mat so t ons

3 Numerical approximation

s s ct on s d ot d to a nonstanda d n t o m m thod to a o mat th so t ons of th non-cons at syst m (38) nc of th s m thod ca d "non n a o ct on m thod" s d sc d n 3 (s a so 6) th sa of s m c ty th s m thod s s nt d n th s s ct on n th cont t of th - d mod s a n m ca m thods a as d on a t o st s s tt ng m thod

Convection s d n d y th syst m

$$\begin{cases} \partial_t \mathbf{w}^C + \partial_x \mathbf{f}(\mathbf{w}) = 0, \\ \partial_t \mathbf{w}^{NC} + \partial_x \mathbf{g}(\mathbf{w}) = Q(\mathbf{w}, \rho s_{n_p}) \left\{ \partial_t \rho s_{n_p} + \partial_x \rho s_{n_p} u \right\}, \\ \mathbf{w}(t = 0, .) = \mathbf{w}^n. \end{cases} \quad (3)$$

t s so d y a non n a o ct on m thod t s m o tant to not that th s non n a o ct on oc d can a d to any hy o c syst m n th fo m (3) nc of th s m thod s as d on a t o st s tt ng t chn q

- *Time evolution.* g n $\mathbf{w}^n$ th fo o ng cons at syst m s a - o mat d th $\mathbf{w}^n$ as n t a data

$$\begin{cases} \partial_t \mathbf{w}^C + \partial_x \mathbf{f}(\mathbf{w}) = 0, \\ \partial_t \mathbf{w}^{NC} + \partial_x \mathbf{g}(\mathbf{w}) = 0, \\ \mathbf{w}(t = 0, .) = \mathbf{w}^n. \end{cases} \quad (40)$$

At th nd of th s st st o ta n a d ct on d not d $\mathbf{w}^{n+\frac{1}{3}}$

- *Nonlinear projection.* In th s co ct on st th a a s $\mathbf{w}^{n+\frac{2}{3}}$ com- t d n th o s st a s d and th nt o y a anc q a- t ons a nfo c d

$$\left\{\begin{array}{c} \partial_t \mathbf{w}^C = 0, \\ \partial_t \mathbf{w}^{NC} + \partial_x \mathbf{g}(\mathbf{w}) = Q(\mathbf{w}, \rho s_{n_p}) \left\{ \partial_t \rho s_{n_p} + \partial_x \rho s_{n_p} u \right\}, \\ \mathbf{w}(t=0,.) = \mathbf{w}^{n+\frac{1}{3}}. \end{array}\right.$$

L t s m has z that th non n a o ct on oc d nfo c s th cons st ncy t n th non-cons at t ms and th n m ca a - o mat ons [illegible] n m ca a o mat on of th non-cons at od- cts s th s f f om th n m ca scos ty and th d sc t fo m of th d s on

Diffusion and source terms a ta n nto acco nt y so ng th syst m

$$\left\{\begin{array}{l} \partial_t \mathbf{w}^C = D(\mathbf{w}) + S^C(\mathbf{w}), \\ \partial_t \mathbf{w}^{NC} = S^{NC}(\mathbf{w}, \rho s_{n_p}), \\ \mathbf{w}^C(t=0,.) = \mathbf{w}^{n+\frac{2}{3}}, \end{array}\right. \qquad (4)$$

h $\mathbf{w}^{n+\frac{2}{3}}$ s th so t on aft th non n a o ct on At th nd of th s st ha com t d $\mathbf{w}^{n+1}$

In th n t s ct ons g d ta s fo th d nt st s of th n m ca a o mat on

3.1 Convection step: The 1-D case

In o d to so th syst m (3) on can s an act o an a o mat d od no so 3 a a at on sch m 6 o any oth n m ca so In th s nt ana ys s th nt o y n q a t s a o ta n d n th cas of an act od no sch m fo a - d m t

cons d a st ct d m sh n s ac and t m d n d y th c s $I_i = (x_{i-\frac{1}{2}}, x_{i+\frac{1}{2}})$ and th t m nt a s $t^n, t^{n+1})$

$$t^n = n\Delta t \quad \text{and} \quad x_{i+\frac{1}{2}} = (i + \frac{1}{2})\Delta x,$$

h Δt s th t m st and Δx th c s ngth [illegible] a o mat d so t on at t m t^n constant n ach c s I_i d not y $\mathbf{w}_i^n$ th a o mat a at t m t^n n c I_i of th a a $\mathbf{w}$ [illegible] n m ca so t on $\mathbf{w}_h^n(x) = \mathbf{w}_h(x, t^n)$ s th n d n d y

$$\mathbf{w}_h^n(x) = \mathbf{w}_i^n \quad \text{h n} \quad x \in I_i.$$

nd th C L cond t on

$$\frac{\Delta t}{\Delta x} \text{ma} \ |\lambda_i(\mathbf{w})| \leq \frac{1}{2}, \qquad (42)$$

th so t on of th Ca chy o m of th syst m (40) th th n t a data $\mathbf{w}_h^n(x)$ s com os d y th so t ons of non nt act ng m nta y mann o ms at th c s nt fac s L t s d not y $\mathbf{w}_{i+\frac{1}{2}}(\xi)$ th act so t on of th m nta y mann o m c nt d at $x_{i+\frac{1}{2}}$

$$\mathbf{w}_{i+\frac{1}{2}}(\xi) = W\left(\xi, \mathbf{w}_i^n, \mathbf{w}_{i+1}^n\right) \quad \text{th} \quad \xi = \frac{x - x_{i+\frac{1}{2}}}{t - t^n}.$$

od no m thod s o ta n d y th o ct on of th so t on com os d of m nta y mann o ms on th s ac of c s constant f nct ons on th c s s s ach d y an a ag ng o ach c L t s d n th n m ca y

$$\phi^{\mathbf{w}}_{i+\frac{1}{2}} = {}^t\left(\mathbf{f}(\mathbf{w}_{i+\frac{1}{2}}(0)), \mathbf{g}(\mathbf{w}_{i+\frac{1}{2}}(0))\right),$$

n th n m ca sch m n th cons at st st s

$$\mathbf{w}_i^{n+\frac{1}{3}} = \mathbf{w}_i^n - \frac{\Delta t}{\Delta x}\left(\phi^{\mathbf{w}}_{i+\frac{1}{2}} - \phi^{\mathbf{w}}_{i-\frac{1}{2}}\right). \tag{43}$$

con nt o y of th syst m (40) $\{\rho s_2\}(\mathbf{w}_i^{n+\frac{1}{3}})$ sat s s a d sc t nt o y n q a ty

$$\{\rho s_2\}(\mathbf{w}_i^{n+\frac{1}{3}}) - (\rho s_2)_i^n + \frac{\Delta t}{\Delta x}\left(\phi^{\rho s_2}_{i+\frac{1}{2}} - \phi^{\rho s_2}_{i-\frac{1}{2}}\right) \leq 0, \tag{44}$$

h

$$\phi^{\rho s_2}_{i+\frac{1}{2}} = \phi^{\rho}_{i+\frac{1}{2}} \tilde{s}_2(\mathbf{w}_{i+\frac{1}{2}}(0)).$$

Mo o th os t n ss of $(e_1)_i^{n+\frac{1}{3}}$ and $(e_2)_i^{n+\frac{1}{3}}$ s ns d as soon as th d ns ty $\rho_i^{n+\frac{1}{3}}$ s os t

n g n a th at of th nt o y d ss at on assoc at d to ρs_2 s st ct y n ga t By th ns n n q a ty ha

$$\{\rho s_2\}(\mathbf{w}_i^{n+\frac{1}{3}}) \leq \frac{1}{\Delta x}\int_{x_{i-1/2}}^{x_{i+1/2}} \{\rho s_2\}(\mathbf{w})(x, t^{n+1})dx. \tag{45}$$

s m ans that th d ss at on of th nt o y $\{\rho s_2\}$ s st ct y n gat On th oth hand th s c c nt o s s_1 and s' a s m y ad ct d y th o and th fo a s d y th c ass ca (L^2) o ct on st At th d sc t th s d sc ancy s ts ca s th fa of th nt o y a anc q at ons (30) and (32) n th s cond st th nt o y d ss at on s d st t n o d to nfo c th a anc (30) and (32) fo $(\rho s_1)_i^{n+\frac{2}{3}}$ and $(\rho s')_i^{n+\frac{2}{3}}$ a com t d f om a d sc t a o mat on of th nt o y a anc q at ons

$$\frac{\beta_i^{n+\frac{1}{3}}}{(T_2)_i^{n+\frac{1}{3}}}\left(\{\rho s_1\}(\mathbf{w}_i^{n+\frac{2}{3}}) - (\rho s_1)_i^{n+\frac{1}{3}}\right) - \frac{1-\beta_i^{n+\frac{1}{3}}}{(T_1)_i^{n+\frac{1}{3}}}\left((\rho s_2)_i^{n+\frac{2}{3}} - (\rho s_2)_i^{n+\frac{1}{3}}\right) = 0, \quad (46)$$

$$\frac{(\lambda_2\mu_2)_i^{n+\frac{1}{3}}}{(T_2)_i^{n+\frac{1}{3}}}\left(\{\rho s'\}(\mathbf{w}_i^{n+\frac{2}{3}}) - (\rho s')_i^{n+\frac{1}{3}}\right) - (\mu')_i^{n+\frac{1}{3}}\frac{\gamma'-1}{(\rho_i^{n+\frac{1}{3}})^{\gamma'-1}}\left((\rho s_2)_i^{n+\frac{2}{3}} - (\rho s_2)_i^{n+\frac{1}{3}}\right) = 0, \quad (4\ )$$

h

$$(\rho s_2)_i^{n+\frac{1}{3}} = (\rho s_2)_i^n - \frac{\Delta t}{\Delta x}\left(\phi^{\rho s_2}_{i+\frac{1}{2}} - \phi^{\rho s_2}_{i-\frac{1}{2}}\right).$$

a o non n a o m n th n no n $(\mathbf{w}^{NC})_i^{n+\frac{2}{3}}$ can s o n to adm t a n q so t on as soon as th a o mat mann so n o d n th st st o ys d sc t nt o y n q a ty fo th La a $(\rho s_2, \rho s_2 u)$ s n t n n q y d n s $(\rho s_2)_i^{n+\frac{2}{3}}$ acco d ng to

$$(\rho s_2)_i^{n+\frac{2}{3}} = \{\rho s_2\}\left(\mathbf{w}_i^{n+\frac{2}{3}}\right).$$

n add t on ha (s 3 fo th oof)

THEOREM 3.1 L t s cons d th sch m (43) nd th C L st ct on (42) th fo o ng d sc t nt o y n q a t s a sat s d

$$\{\rho\,\psi_\ell(s_\ell)\}(\mathbf{w}_i^{n+\frac{2}{3}}) - (\rho\,\psi_\ell(s_\ell))_i^n + \frac{\Delta t}{\Delta x}\left\{\{\rho\,\psi_\ell(s_\ell)u\}_{i+1/2}^n - \{\rho\,\psi_\ell(s_\ell)u\}_{i-1/2}^n\right\} \leq 0, \quad \ell = 1, 2,$$

fo any st ct y nc as ng f nct ons ψ_ℓ ass m d to sat sfy th con ty of th ma s $\mathbf{w} \to \rho\,\psi_1(s_1)$ and $\mathbf{w} \to \rho\,\psi_2(s_2(\mathbf{w}))$ fo o ng ma m m nc s fo th s c c nt o s a m t

$$(s_\ell)_i^{n+\frac{2}{3}} \leq \max\left((s_\ell)_{i-1}^n, (s_\ell)_i^n, (s_\ell)_{i+1}^n\right), \quad \ell = 1, 2. \quad (48)$$

a t a s c c nt na n g s $(e_1)_i^{n+\frac{2}{3}}$ and $(e_2)_i^{n+\frac{2}{3}}$ stay os t as soon as th d ns ty $\rho_i^{n+\frac{2}{3}}$ s os t and th ma m m nc s $0 \leq (Y_{1,2})_i^{n+\frac{2}{3}} \leq 1$ a sat s d

th m t d m ns ona cas s on y th *time evolution* st s d nt f om th cas o th n m ca so t a n d y a t nd d at nt fac s t n c s

3.2 Diffusion and source terms

In th s nt a do not d o th d sc t fo m aton of th d s o ato and f th ad to 5 o (and th f nc s th n) h s a n m ca m thods a o os d Conc n ng th so c t ms ass m that th s z of so c t ms s sma com a d to dynam c of th o (go n d y th hy o c syst m) h fo th n m ca a o mat on s ach d y a s tt ng t ch q C a chy o m so d n th add t ona st s

$$\begin{cases} \partial_t \rho = 0, \quad \partial_t \rho u = 0, \quad \partial_t \rho Y = 0, \\ \partial_t \rho_\ell e_\ell = \rho_\ell \epsilon, \\ \partial_t \rho k = -\rho\epsilon, \\ \partial_t \rho\epsilon = -C_2 \rho \dfrac{\epsilon^2}{k}. \end{cases}$$

h s syst m s nt g at d ana yt ca y th th n t a a $\mathbf{w}^{n+\frac{2}{3}}$ to o ta n

$$\begin{cases} \rho^{n+1} = \rho^{n+\frac{2}{3}}, \quad u^{n+1} = u^{n+\frac{2}{3}}, \quad Y^{n+1} = Y^{n+\frac{2}{3}}, \\ k^{n+1} = \left(\dfrac{(k^{n+\frac{2}{3}})^{C_2}}{(C_2 - 1)\epsilon^{n+\frac{2}{3}}\Delta t + k^{n+\frac{2}{3}}} \right)^{\frac{1}{C_2-1}}, \\ \epsilon^{n+1} = \dfrac{\epsilon^{n+\frac{2}{3}} k^{n+1}}{(C_2 - 1)\epsilon^{n+\frac{2}{3}}\Delta t + k^{n+\frac{2}{3}}}, \\ e_\ell^{n+1} = e_\ell^{n+\frac{2}{3}} + (k^{n+\frac{2}{3}} - k^{n+1}), \quad 1 \le \ell \le n_p. \end{cases}$$

h fo th n m ca t m st s com t y d n d

3.3 The 2-D extension

h m t d m ns ona t ns on do s not n o a g d c t s c t d th stand a d o ms t h n a o mat ng o a -Sto s q at ons h 2-D syst m s g n y

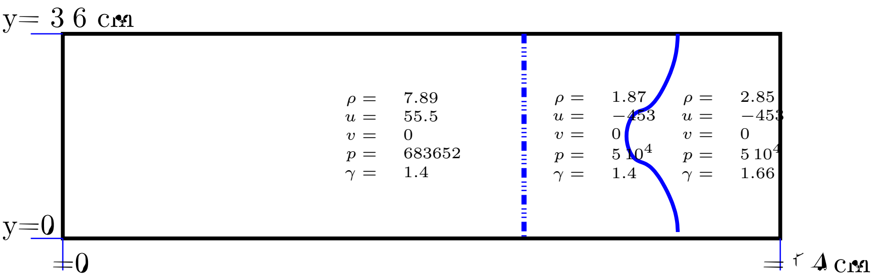
$$\begin{cases} \partial_t \mathbf{w}^C + \partial_x \mathbf{F}_1(\mathbf{w}) + \partial_y \mathbf{F}_2(\mathbf{w}) = D(\mathbf{w}) + S^C(\mathbf{w}), \\ \partial_t \mathbf{w}^{NC} + \nabla\cdot(\mathbf{G}(\mathbf{w})) = S^{NC}(\mathbf{w}, \rho s_{n_p}) + Q(\mathbf{w}, \rho s_{n_p}) \left\{ \partial_t \rho s_{n_p} + \nabla\cdot\left(\rho s_{n_p} \mathbf{u}\right) \right\}, \end{cases}$$

h

$$\mathbf{w}^C = \begin{pmatrix} \rho \\ \rho\mathbf{u} \\ E \\ \rho Y_1 \\ \frac{\rho k^{C_1}}{\epsilon} \end{pmatrix}, \quad \mathbf{F}_1(\mathbf{w}) = \begin{pmatrix} \rho u_1 \\ \rho u_1^2 + p + p' \\ \rho u_1 u_2 \\ (E + p + p')u_1 \\ \rho Y_1 u_1 \\ \frac{\rho k^{C_1}}{\epsilon} u_1 \end{pmatrix}, \quad \mathbf{F}_2(\mathbf{w}) = \begin{pmatrix} \rho u_2 \\ \rho u_1 u_2 \\ \rho u_2^2 + p + p' \\ (E + p + p')u_2 \\ \rho Y_1 u_2 \\ \frac{\rho k^{C_1}}{\epsilon} u_2 \end{pmatrix},$$

and

$$\mathbf{u} = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}, \quad \mathbf{G}(\mathbf{w}) = \begin{pmatrix} \rho s_1 \mathbf{u} \\ \\ \rho s_{n_p-1}\mathbf{u} \\ \rho s' \mathbf{u} \end{pmatrix}.$$



ꟹg ꟹn t a zat on of th non ꟹ ct at ng a a s

Onc aga n ado t a s tt ng t chn q do not d t a th n m ca a o-mat on of th d s on o ato and th so c t m h ch m t a s a fo m (s o) fo o ng th ca foc s o att nt on on th con ct on st d at ng fo m a (43) to o n t m th n no n cto w s no g n y

$$\mathbf{w}_i^{n+\frac{1}{3}} = \mathbf{w}_i^n - \frac{\Delta t}{a_i} \sum_{j \in \mathcal{V}(i)} \phi_{ij}^{\mathbf{w}},$$

h a_i s th a a of th cont o o m $\mathcal{V}(i)$ d not s th s t of th n gh o ng c s to c i [redacted] n m ca f nct on $\phi_{ij}^{\mathbf{w}}$ s com t d f om th act o a o mat so t on of th m nta y mann o m s tat d at th c nt fac

$$\phi_{ij}^{\mathbf{w}} = \phi^{\mathbf{w}}(\mathbf{n}_{ij}, \mathbf{w}_i^n, \mathbf{w}_j^n),$$

h $\mathbf{n}_{ij}$ s th o t n t no ma to th c nt fac t n c s i and j [redacted] s cond st of th s tt ng na m y th non n a o ct on ma ns g n y (46)-(4) t fo th fo o ng d n t on of $(\rho s_2)_i^{n+\frac{1}{3}}$

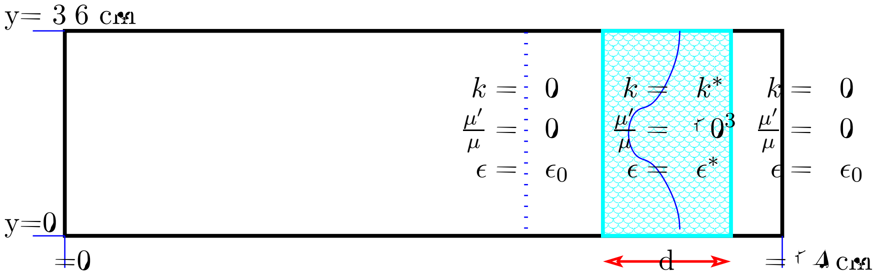
$$(\rho s_2)_i^{n+\frac{1}{3}} = (\rho s_2)_i^n - \frac{\Delta t}{a_i} \sum_{j \in \mathcal{V}(i)} \phi_{ij}^{\rho s_2}.$$

[redacted] t ns on of th sch m to m t-d m ns on s th s ach d

4 Numerical results

cons d n th s s ct on th n m ca com tat on of a 2 chtm y -M sh o nsta ty [redacted] nsta ty s d o d y th nt act on of a s oc a th a mat a nt fac t n t o non m ng ꟹ ds ass m that th ꟹ d com o-n nts a f ct gas and that th ꟹ ct at ons a at th t nc sca [redacted] fo can s th fo o ng mod ng constants $C_1 = .4$ and $C_2 = o$ (s o)

[redacted] d sc t sch m s fo m at d on nst ct d t ang a m s h s and th cont o o m s a of *cell vertex* ty (s onga 20) [redacted] n m ca ꟹ s at c s nt fac s a com t d y th a at on sch m o os d n 6 [redacted] acc acy of



g 2 n t a zat on of th ct at ng a a s

th a o mat on s m o d y a ng - tta s cond o d t m a o mat ons and s cond o d s ac a o mat ons as d on a M SCL ty t chn q

com tat ona doma n s $0, 0. 4m \times 0, 0.036m$ co d y an nst c- t d t ang at on mad of 80000 t ang s th 4050 t c s ch a act st c s z s of th m sh a $\Delta x \simeq \frac{0.14}{400}$ and $\Delta y \simeq \frac{0.036}{100}$ th C n m s d to 0 5 fo a th com tat ons

n t a y th t o com on nts of th d a s a at d y a osc at ng c nt fac ocat d at $x = 0. 2$ and of s z 0.005 d n d y (s 6)

$$x - 0. 2 = 0.005 \cos\left(\frac{2\pi(y - 0.0 8)}{0.036}\right)$$

s nt fac c oss d y a shoc n t a y ocat d at $x = 0.0 \ m$ assoc at d to th ft stat g n y $\rho = .8 \ kg/m^3$ $P = 68365 2 \ Pa$ $u = 55.5 \ m/s$ and th shoc a oc ty $\sigma = 2 3.5 \ m/s$ ad s a so f d to th o of Lo s h s m a n m ca m nts a fo m d

Com tat ons a fo m d fo d nt s z s (d) of th ct at ng zon a o nd th nt fac and fo d nt a s of th ct at ng n t c n gy (k^*) d nt t sts cas fo m d h a d n d y

st cas 0			$d = 0$
st cas A	$k^* = 0 \, Pa$	$\epsilon^* = 8$	$d = 5mm$
st cas B	$k^* = 0 \, Pa$	$\epsilon^* = 8$	$d = 30mm$
st cas C	$k^* = 30000 \, Pa$	$\epsilon^* = 6200000$	$d = 5mm$

n m ca nt fac s o ta n d y th f act on β d n d y (3) m ca s ts g a ha o of th chtm y -M sh o nsta ty that s acc at d and mo d d h n ct at ons a cons d d (g 4) nd d th o at th t m $t = 2.0 \ ms$ h n th s no ct at on s com a a to th o o ta n d at th t m $t = .6 \ ms$ th an n t a ct at ng zon a o nd th mat a nt fac (s g 3) s mod cat on s o y d nds on th n t a ct at ons zon s z o th ct at on com tat ons o ta n d fo th t st cas s A B and C a y c os (s g 5)

5 Conclusion

nd som hys ca ass m t ons ha d d a s m fy mod fo m t- d o ta ng nto acco nt th n nc of oc ty ct at ons mod s c os to th c ass ca t nc mod t s f nda m nta y non cons at t s as- soc at d to som nt o y n q a t s Bas d on th non n a o ct on ha d o d a n m ca a o mat on cons st nt th th ma n o t s of th mod m ca com tat ons ha o nt o t th m o tanc of th oc ty c- t at ons on th d o m nt of th chtm y -M sh o n ta ty y d nt ha o s a o ta n d h n ct at ons a cons d d o th g o a - ha o so y d nd nt on th s z of th n t a ct at ng zon and th of ct at ons cont o th oc ty of th nsta t s

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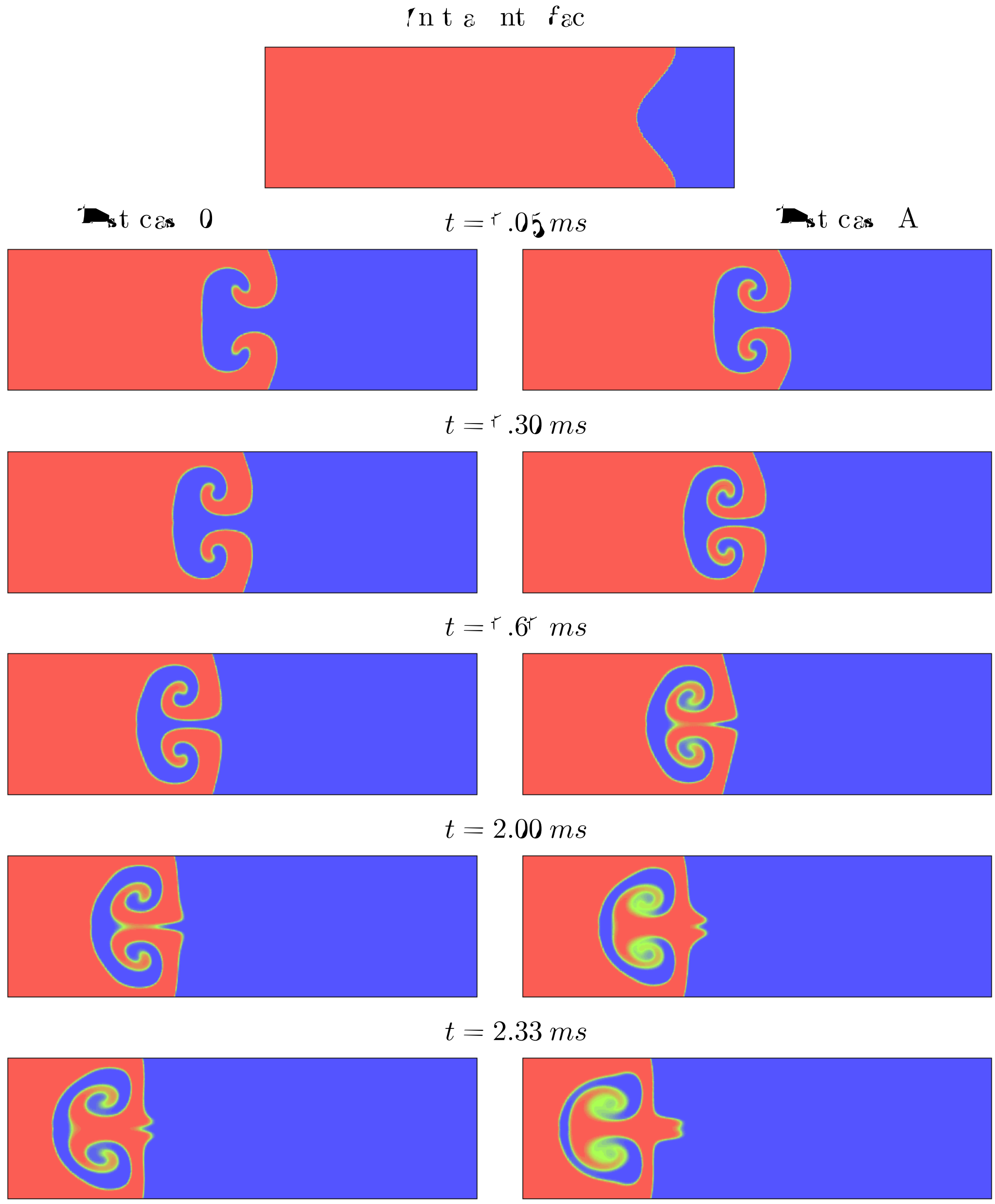
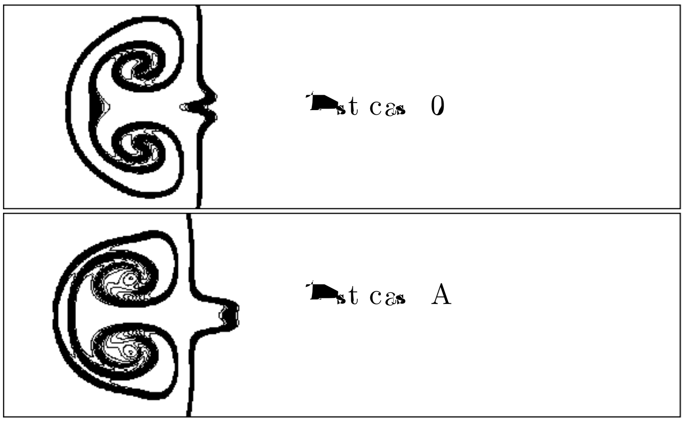


Fig. 3 Influence of the interactions on the behavior of the Richtmyer-Meshkov instability: time evolution of a color function solution on the (right) and on the (left) an interacting zone around the material interface flows (defined by the same color function) at the times $t = 0$, $t = 1.05\,ms$, $t = 1.3\,ms$, $t = 1.67\,ms$, $t = 2.0\,ms$, $t = 2.33\,ms$



g 4 Com a son of th n m ca mat a nt fac o s (d n d y th sam co o f nct on) at th t m $t = 2.33\,ms$, t n com tat ons fo m d th o t (t cas 0), and th (t cas A) an n t a ct at ng zon a o nd th mat a nt fac

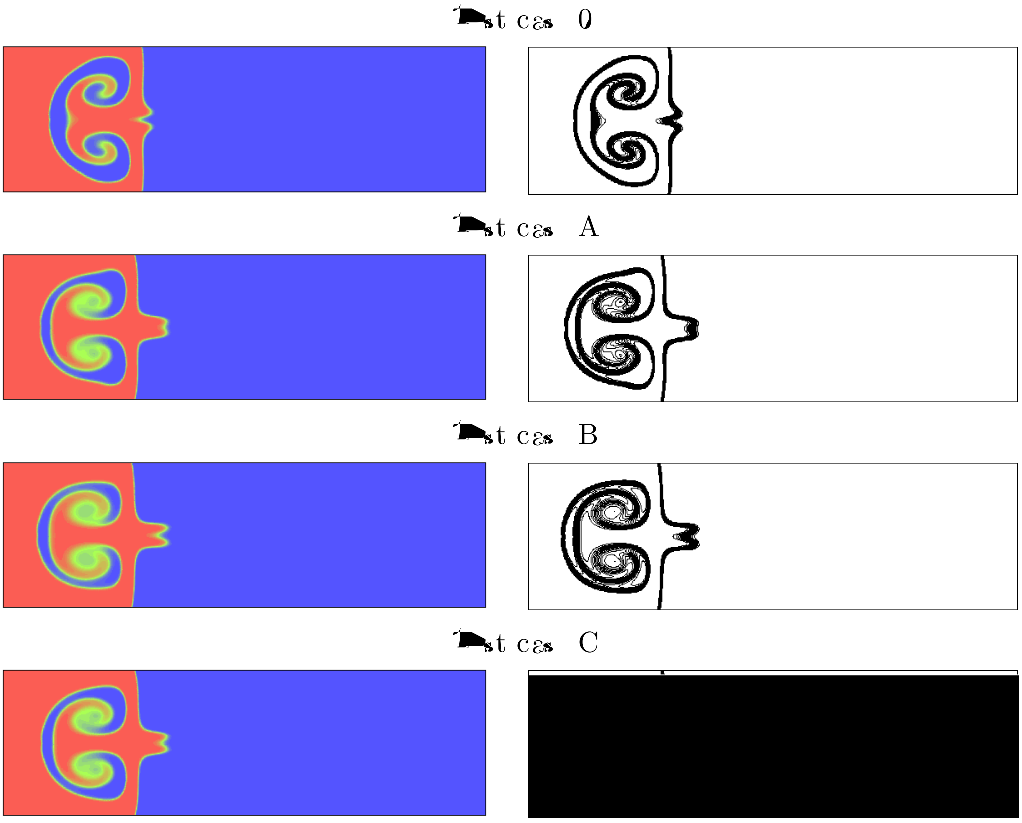


Fig. 5: Effects of the initial perturbating conditions on the Richtmyer-Meshkov instability. Isolines (defined by the same color function) at the time $t = 2.33\,ms$

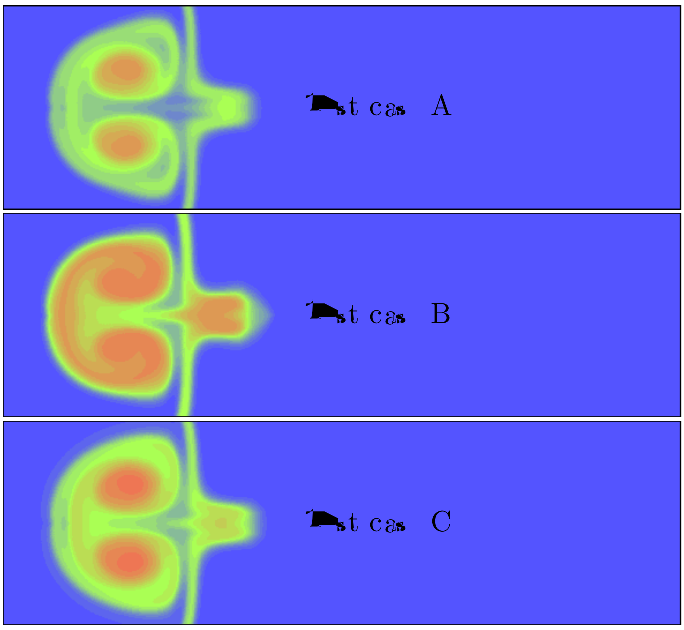


Fig 6 ct at ng n t c n gy ρk at t m $t = 2.33\,ms$ o s fo th t st cas s A B and C